Time to Dump the Rectangular Stress Block?

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Synopsis: Since the early days of ultimate strength design of reinforced and prestressed concrete the concrete stress distribution has commonly been approximated with a rectangular stress block, applied over a specified proportion of the zone of concrete in compression. This provides a simple design procedure, and for the concrete strength grades used in the past it provides a good level of accuracy. With the release of the current version of the Australian Standard Concrete Structures Code (AS 3600 – 2009) it has been necessary to modify the parameters of the rectangular stress block for concrete strength grades of 65 MPa and higher, because of their higher strain before reaching yield, and much reduced stress plateau after yield. This paper compares the results of the revised rectangular stress block with those from the main international codes, and with stress-strain distributions closer to the actual behaviour of high strength concrete. Both rectangular and non-rectangular sections are considered. These alternatives are compared for consistency, reliability, and ease of use, and recommendations are made for the appropriate use of the rectangular stress block or other less simplified alternatives in a design context.

Keywords: rectangular stress block, stress-strain, moment capacity, AS 3600, ACI 318, Eurocode 2, interaction diagram

1. Introduction

The simplification of the compressive stress distribution in a reinforced concrete section by a rectangular stress block (Figure 1) is widely accepted both in Australian codes (e.g. AS 3600 (1), AS 5100 (2)) and by international codes (e.g. ACI 318 (3), Eurocode 2 (4)). The basis of this approach is that by factoring the ultimate concrete stress and the depth of the stress block it is possible to generate a simple stress distribution that has exactly the same total force and lever arm as a more complex distribution that more closely follows the actual stress distribution in a real concrete section. For concrete strengths up to 50 MPa the codes examined in this paper apply a fixed factor to the concrete strength and either a fixed or variable depth of the stress block, based on a rectangular section. This provides a close match to the actual concrete behaviour for rectangular sections, or sections where the neutral axis is within a rectangular top flange, and an acceptable level of accuracy for non-rectangular sections. For higher concrete strengths the strain at yield is higher, and the width of the stress plateau after yield is much reduced (Figure 2). For these reasons it is necessary to adjust the rectangular stress block factors for concrete strengths over 50 MPa, to maintain a close approximation to the actual behaviour of the concrete.

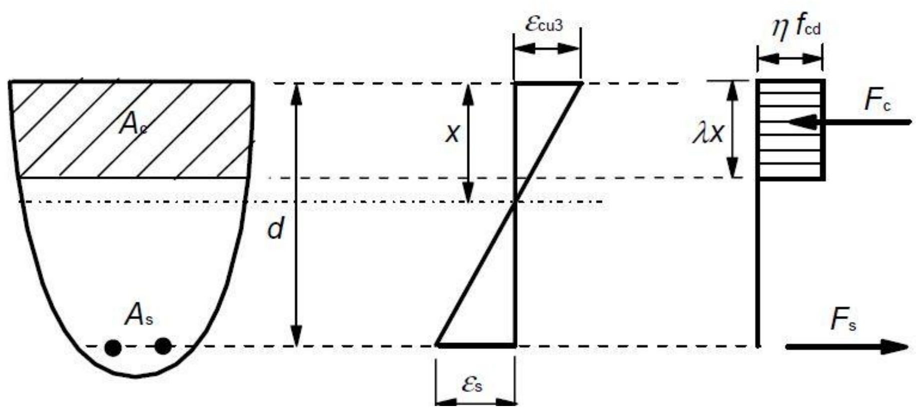

Figure 1. Rectangular stress block (Eurocode 2 Figure 3.5)

2. Code Stress Block Factors

2.1 General

In this paper the results from the current AS 3600 rectangular stress block are compared with the rectangular stress blocks from other codes, the parabolic-rectangular stress block from Eurocode 2 and the concrete stress-strain diagrams shown in Figure 2. The other codes considered are:

- The Australian Bridge Code, AS 5100 (2), and the previous version of AS 3600 (5).
- The American Concrete Institute Building Code for Structural Concrete, ACI 318 (3)
- Eurocode 2, Design of Concrete Structures (4)

For ease of comparison the nomenclature used in AS 3600 is used for the equivalent factors in all the other codes. The rectangular stress block is defined by the following parameters and factors:

- f'_c = characteristic compressive (cylinder) strength of concrete at 28 days
- The maximum strain in the extreme compression fibre, here designated ε_{ur}
- α_2 = stress reduction factor accounting for both the lower long term strength of concrete under sustained load (AS 3600 Cl 3.1.4), and the variation of the actual concrete stress from the assumed rectangular block (AS 3600 Cl 8.1.3 (b)). This factor is in addition to any Ultimate Limit State (ULS) reduction factors or partial safety factors.
- The limit of the stress block, defined by a line parallel to the neutral axis under the loading concerned, and located at a distance $\gamma k_u d$ from the extreme compressive fibre, where $k_u d$ is the depth of the neutral axis from the extreme compression fibre.

Where different symbols are used in other codes these are stated when the factor is first defined.

The various codes also have significantly different provisions for application of ULS capacity and stress reduction factors, but these are outside the scope of this paper.

2.2 AS 3600 and 5100; Australian Standards for Concrete Buildings and Bridges

The provisions of AS 5100 and the previous version of AS 3600 (AS 3600 – 2001 (5)) are identical for the provisions relating to the rectangular stress block:

- $\varepsilon_{ur} = 0.003$
- $\alpha_2 = 0.85$ (no symbol was given for this factor)
- $\gamma = [0.85 - 0.007\,(f'_c - 28)]$ within the limits 0.65 to 0.85 (symbol used = γ)
- The maximum concrete grade covered by the codes = 65 MPa.

In the current AS 3600 (as revised in Nov 2010) these factors have been adjusted to allow for concrete grades up to 100 MPa:

- $\alpha_2 = 1.0 - 0.003\, f'_c$ (within the limits $0.67 \le \alpha_2 \le 0.85$)
- $\gamma = 1.05 - 0.007\, f'_c$ (within the limits $0.67 \le \gamma \le 0.85$)

Note that the new provision for α_2 only affects concrete with a cylinder strength greater than 50 MPa, and the provision for γ is almost unchanged from the previous version because the minimum value of 0.67 is applicable to any concrete with a grade greater than 55 MPa.

2.3 ACI 318; American Concrete Institute Building Code for Structural Concrete

The provisions of ACI 318 are very similar to AS 5100:

- $\varepsilon_{ur} = 0.003$
- $\alpha_2 = 0.85$ (no symbol given)
- $\gamma = [0.85 - 0.05\,(f'_c - 28)/7]$ within the limits 0.65 to 0.85 (symbol used = β_1)
- No maximum concrete grade is specified

2.4 Eurocode 2, Design of Concrete Structures

Eurocode 2 has detailed provisions for either a rectangular, a linear-rectangular, or a parabolic–rectangular stress block. The code provides recommendations for all factors, but some may be specified within the National Annex of the country where the code is used. In the case of the α factor the equivalent factor in Eurocode 2 is $\alpha_{ct}.\eta$, where α_{ct} has a recommended value of 1.0, but the UK National Annex value is 0.85. For the purposes of this paper a value of 0.9 has been used for consistency with the AS 3600 reduction on concrete cylinder strength specified in Clause 3.1.4.

- ε_{ur} = 0.0035 for concrete to grade 50, 0.0026+ 0.035[(90-f'_c)/100]^4 for grades over 50 MPa (symbol ε_{cu3})
- α_2 = 0.9 for concrete to grade 50, 0.9(1.0 – (f'c – 50)/200) for grades over 50 MPa (symbol $\alpha_{ct}.\eta$)
- γ = 0.8 – (f'_c – 50)/200] within the limits 0.60 to 0.80 (symbol used = λ)
- The maximum concrete grade covered by the code = 90 MPa.

For the parabolic-rectangular stress block Eurocode 2 defines the stress as

$$\sigma_c = f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] \quad \text{for } 0 \le \varepsilon_c \le \varepsilon_{c2} \qquad (1)$$

Where f_{cd} is the design compressive strength, taken here as 0.9f'_c, and ε_{c2} is the strain at reaching maximum stress.

3. Concrete Stress-Strain Relationships

AS 3600 states that "The stress-strain curvature for concrete shall be (either) assumed to be of curvilinear form defined by recognized simplified equations … (Cl. 3.1.4)(1). For the purposes of this paper the simplified stress blocks specified in the codes have been compared with the results of using the stress-strain relationships published by Collins et al. (6) and that specified in Eurocode 2 for non-linear structural analysis (C. 3.1.5)(4).

The curve proposed by Collins et al. has the same form for ascending and descending stress, but the coefficient varies for the descending part, depending on the concrete strength:

$$\frac{\sigma_c}{f'_c} = \left(\frac{\varepsilon_c}{\varepsilon_{c0}}\right)\frac{n}{n-1+\left(\frac{\varepsilon_c}{\varepsilon_{c0}}\right)^{nk}} \qquad (2)$$

$$\varepsilon_{c0} = \left(\frac{f_c}{E_{ct}}\right)\frac{n}{n-1} \qquad (3)$$

where k = 1 for $\varepsilon_c \le \varepsilon_{co}$, k = 0.67 + ($f_c$/62) for $\varepsilon_{co} \le \varepsilon_c$, n = 0.8 + ($f_c$/17) and E_{ct} = 3320 $(f_c)^{0.5}$ + 6900, with f_c in MPa.

The Eurocode 2 curve has the same form and coefficients for both ascending and descending stress:

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta-\eta^2}{1+(k-2)\eta} \qquad (4)$$

Where: $\eta = \varepsilon_c/\varepsilon_{c1}$, ε_{c1} is the strain at peak stress and k = 1.1 $E_{cm} \times |\varepsilon_{c1}|/f_{cm}$

ε_{c1} is given by: 0.0007 $f_{cm}^{0.31}$ < 0.0028

$E_{cm} = 22000(f_{cm}/10)^{0.3}$ MPa

The equation is valid up to the nominal ultimate strain, ε_{cu1}, equal to 0.0035 for cylinder strengths less than 50 MPa, and given by: $0.0028 + 0.027((98 – f_{cm})/100)^4$

f_{cm} in Eurocode 2 is defined as the mean value of the concrete cylinder compressive strength, and is given by $f_{cm} = f_{ck} + 8$ MPa where f_{ck} is the characteristic compressive cylinder strength of concrete at 28 days.

For the purposes of this paper f_{cm} in Equation 4 has been replaced by f'_c (f_{ck} in Eurocode 2), since the ultimate strength is based on the characteristic compressive strength, rather than the mean strength.

Stress-strain curves using these two equations are shown in Figure 2 for concrete strengths of 30 MPa, 60 MPa and 90 MPa.

3.1 Equivalent Rectangular Stress Block; Rectangular Sections

For a rectangular concrete section, or where the compression zone is entirely within a rectangular section, it is possible to find a rectangular stress block that will result in exactly the same ultimate bending capacity as that derived from any non-linear stress-strain relationship:

- The height of the rectangular stress block must be chosen so that the centroid is in the same position as the centroid of the area under the stress-strain curve.
- The uniform stress for the rectangular stress block must be equal to the average stress under the stress-strain curve.

With a neutral axis depth of x below the extreme compression flange, if the non-linear stress-strain curve centroid is at k.x above the neutral axis, and the average stress is $a.f'_{cu}$, where f'_{cu} is the design maximum concrete compressive stress then:

- The rectangular stress block height factor, γ, = $2(1 – k)$
- The stress factor, α_2. = a/γ

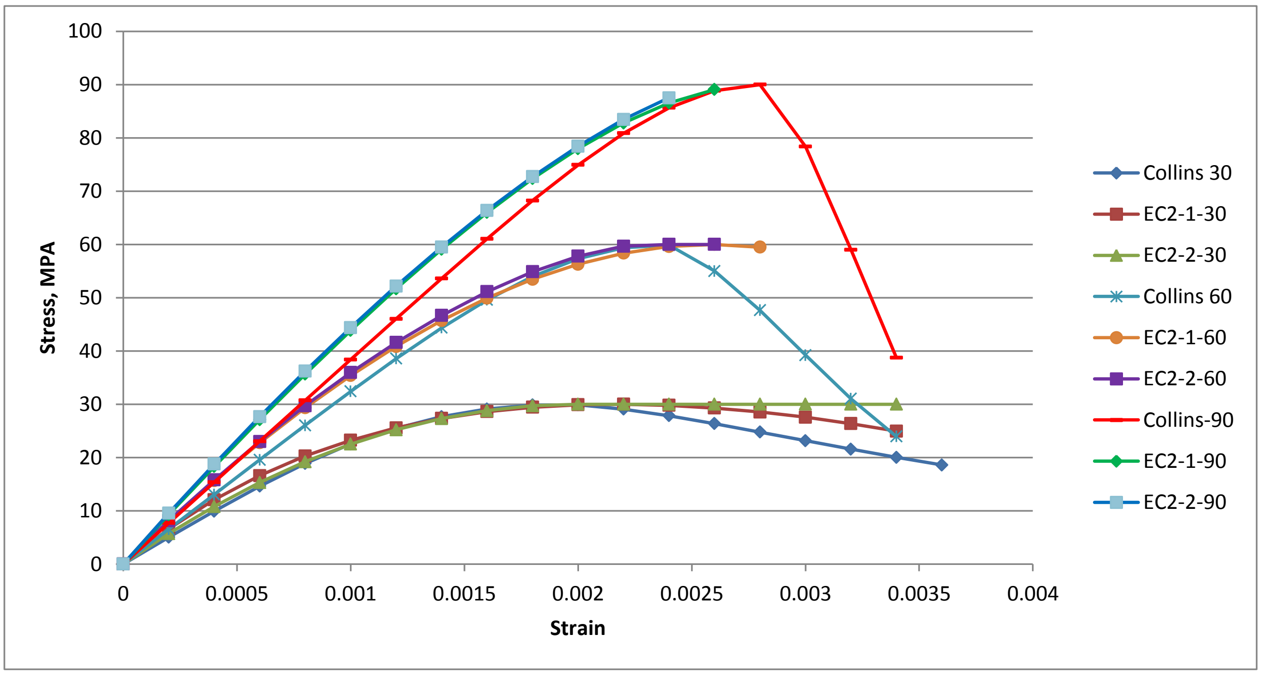

Figure 2. Stress-strain curves for concrete f'_c = 30 MPa, 60 MPa and 90 MPa

3.2 Equivalent Rectangular Stress Block and Code Requirements

Table 1 shows the equivalent rectangular stress block parameters derived from the three non-linear stress-strain curves described above, compared with the parameters specified in AS 3600, Eurocode 2, and ACI 318. Figure 4 shows $\alpha.\gamma$, equal to the normalised average stress, plotted against concrete strength, and Figure 5 shows $(\alpha.\gamma)(1 - \gamma/2)$, equal to normalised stress x lever arm.

Table 1. Code and Equivalent Stress Block Parameters

Concrete Strength fc MPa	Equivalent Stress Block Parameters						Code Rectangular Stress Block Parameters					
	Collins		Eurocode 2 Non-Linear		Eurocode 2 Parabolic		AS 3600		ACI 318		Eurocode 2	
	α	γ	α	γ	α	γ	α	γ	α	γ	α	γ
25	0.833	0.821	0.858	0.840	0.876	0.832	0.850	0.850	0.850	0.850	0.900	0.800
32	0.812	0.806	0.852	0.820	0.876	0.832	0.850	0.826	0.850	0.821	0.900	0.800
40	0.797	0.787	0.844	0.800	0.876	0.832	0.850	0.770	0.850	0.764	0.900	0.800
50	0.788	0.762	0.833	0.780	0.876	0.832	0.850	0.700	0.850	0.693	0.900	0.800
65	0.784	0.725	0.813	0.757	0.804	0.732	0.805	0.670	0.850	0.650	0.833	0.763
80	0.770	0.698	0.791	0.738	0.758	0.709	0.760	0.670	0.850	0.650	0.765	0.725
90	0.749	0.687	0.783	0.728	0.744	0.706	0.730	0.670	0.850	0.650	0.720	0.700

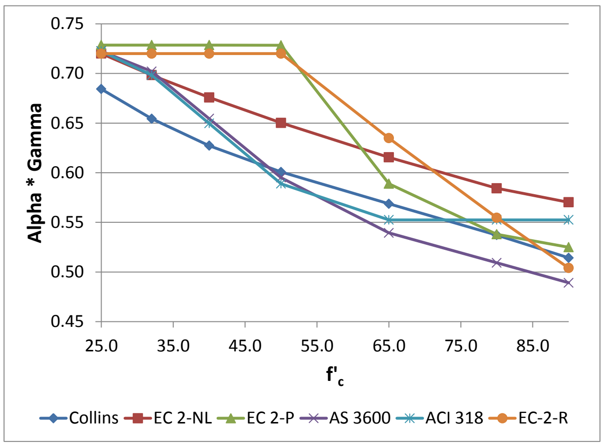

Figure 3. Stress block parameters: α.γ

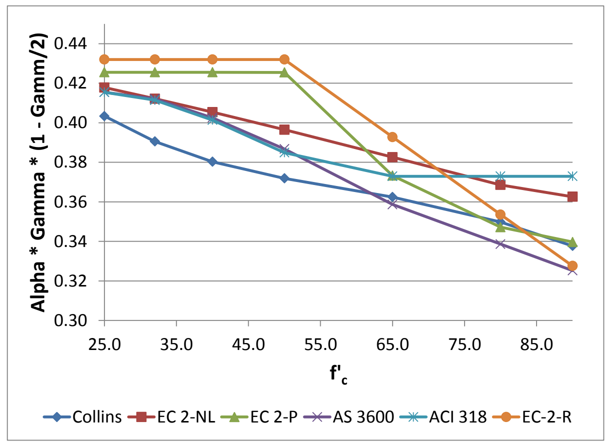

Figure 4. Stress block parameters: (α.γ)(1 - γ/2),

4. Section Capacity

Moment capacity vs. axial load interaction diagrams have been generated for three different cross sections (rectangular, circular, and I-section, see Table 2), with three concrete grades (32 MPa, 65 MPa, and 90 MPA). The following stress-strain curves were used: “Collins” model (Collins); Eurocode 2 non-linear (EC2-NL); Eurocode 2 Parabolic-Rectangular (EC2-P); AS 3600 Rectangular (AS 3600); ACI 318 Rectangular (ACI 318); Eurocode 2 Rectangular (EC2-R); and the rectangular stress block equivalent to EC2-P (Equ-R), as defined in Section 3.1. For consistency with the requirements of AS 3600, and to avoid differences in procedure outside the scope of this paper, the following procedure was adopted:

- Calculations were in accordance with the requirements of AS 3600 Cl. 8.1.2 (Basis of Strength Calculations) and Cl. 3.1.4 (Stress-strain curves).
- No capacity reduction factors or partial safety factors were applied. The calculated moment capacity is therefore the ultimate strength in bending, M_u, in AS 3600 terminology.
- Where the maximum stress of stress-strain curves or stress blocks was based on the concrete cylinder strength this stress was factored by 0.9, in accordance with AS 3600 Cl. 3.1.4
- The maximum stress used for the EC2-NL stress-strain curve was based on the characteristic cylinder strength, f_{ck}, rather than the mean cylinder strength, f_{cm}.
- Reinforcement was taken to have an elastic-plastic stress-strain curve with a yield stress of 500 MPa, and an elastic modulus of 200 GPa.
- The stress of reinforcement in the compression zone was reduced by the stress applied to the concrete at that level, to correct for the area of concrete displaced by the steel.
- The section ultimate bending moment was taken about the centroid of the gross concrete section.

Table 2. Section details

	Rectangular	Circular	I-Section
Dimensions, mm	1000 x 350 deep	600 dia	1000 x 200 400 to 200 x 200 200 x 300 400 to 200 x 200 1000 x 200
Reinforcement	10-N16 Top 10-N20 Bottom	12-N20	10-N16 Top 10-N20 Bottom
Cover, mm	40 Both faces	30	40 Both faces

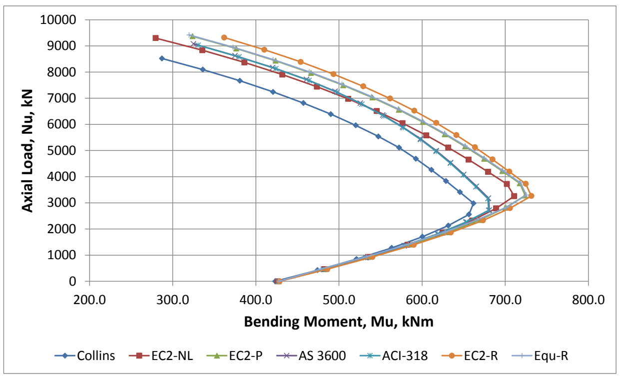

Figure 5: Rectangular Section; 32 MPa

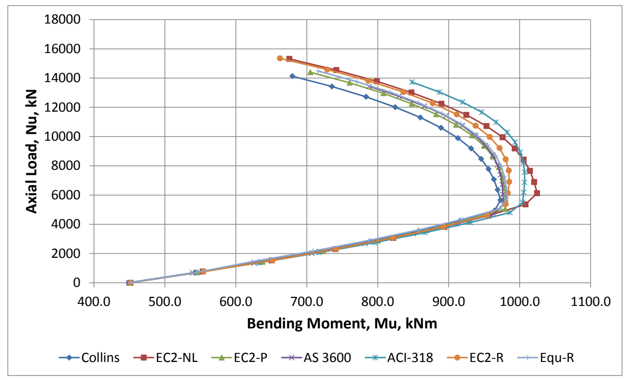

Figure 6. Rectangular Section; 65MPa

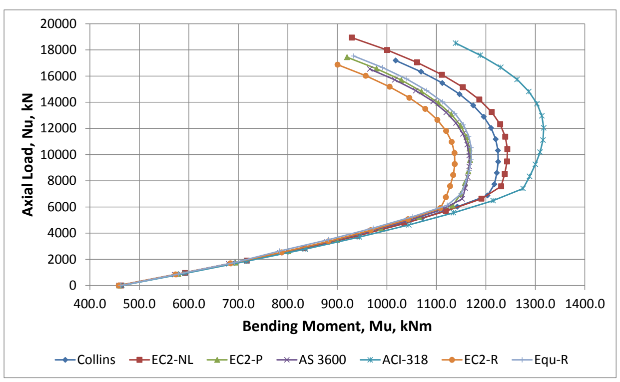

Figure 7: Rectangular Section; 90 MPa

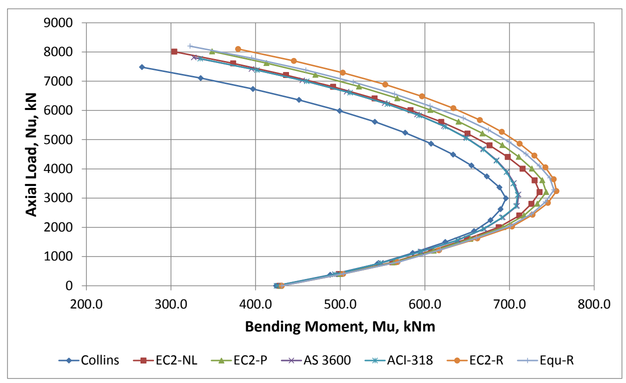

Figure 8. Circular Section;32 MPa

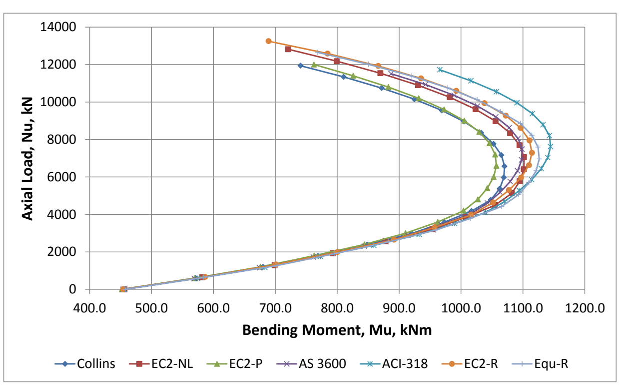

Figure 9. Circular Section; 65 MPa

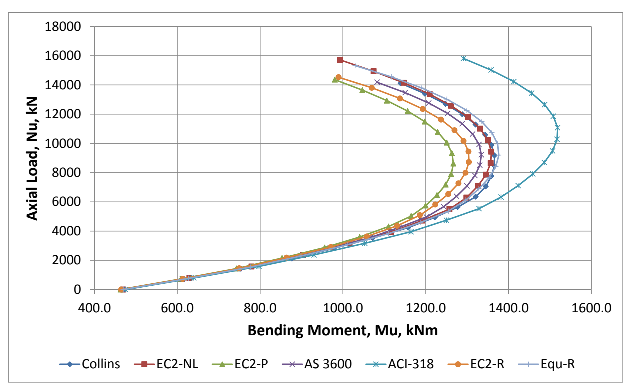

Figure 10. Circular Section; 90 MPa

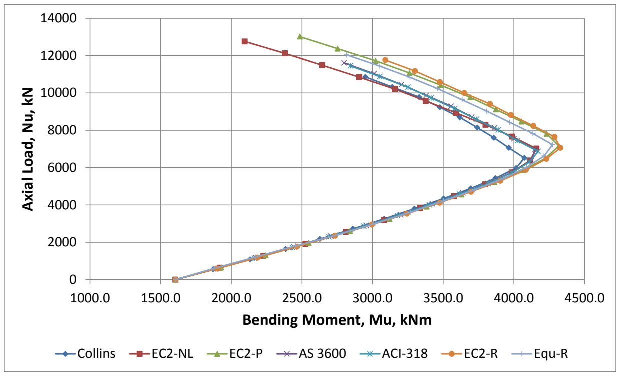

Figure 11. I Section; 32 MPa

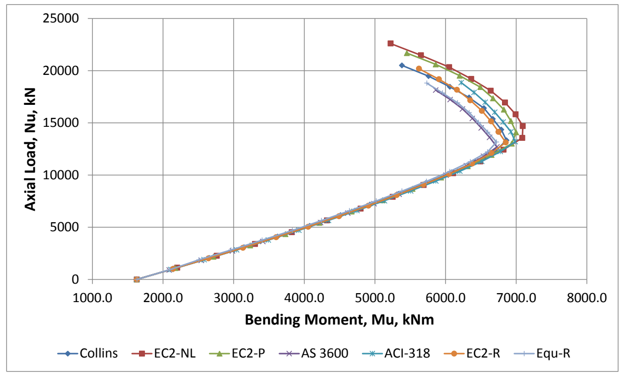

Figure 12. I Section; 65 MPa

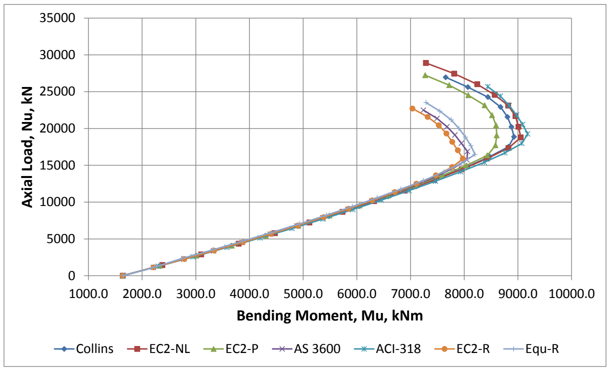

Figure 13. I Section; 90 MPa

- The interaction diagrams were calculated for axial loads between zero and the load at which the neutral axis passed through the base of the section.
- The strain at the compression face (ε_{ur}) used in conjunction with the curvilinear stress-strain curves was chosen to maximise the ultimate bending moment at the balance axial load, in accordance with AS 3600 Cl. 8.1.2 Note 1(b). The value of ε_{ur} used for each concrete strength and section type is shown in Table 3.
- The position of the neutral axis for each load was found using Brent's Method and Numerical Integration for the curvilinear stress-strain curves and the circular sections. For the rectangular stress blocks with sections made up of one or more trapeziums a closed form solution was used, based on solution of the quadratic or cubic equation for the axial load.

Table 3. ε_{ur} for Curvilinear Stress-Strain Curves

	Section Type		
f'c	Rect	Circ	I Sect
32	0.0032	0.0035	0.0025
65	0.0029	0.0031	0.0028
90	0.0030	0.0033	0.0030

The following conclusions may be drawn from these analyses:

- The concrete stress-strain relationship had little effect on the section capacity for axial loads up to about half of the balance load.

- The Collins curve gave progressively increased capacity, relative to the other curves. With the rectangular section it was the most conservative at 32 MPa, slightly conservative at 65 MPa, and the third least conservative at 90 MPa.
- The EC2-NL curve followed the same trend, but from a higher base, being mid-range at 32 MPa, and the least conservative, other than ACI 318, at 90 MPa.
- The EC2-P curve was in the mid-range for all concrete strengths for the rectangular section.
- The AS 3600 stress block was also in the mid-range for all concrete strengths for the rectangular section.
- The ACI 318 stress block was very close to AS 3600 at 32 MPa, but was significantly less conservative than any other curve at 90 MPa, with a bending capacity about 40% higher than the EC2-P curve for the circular section at the balance load.
- The EC2-R stress block was least conservative at 32 MPa and most conservative at 90 MPa (other than for the circular section, see below).
- The Equ-R stress block was almost exactly equivalent to the EC2-P curve, upon which it was based) for the rectangular section, as would be expected. It followed the same trend as the other rectangular stress blocks for the other sections (see below).
- The results from the rectangular stress blocks were increasingly unconservative with increasing concrete strength for the circular section. The difference between the Equ-R bending capacity and the EC2-P capacity at balance load increased from about 1% with 32 MPa concrete to about 9% with 90 MPa concrete.
- The difference between the AS 3600 bending capacity and the EC2-P capacity for the circular section at balance load was about 5% with 90 MPa concrete.
- The rectangular stress blocks were increasingly conservative with increasing concrete strength for the I-section. The difference between the Equ-R bending capacity and the EC2-P capacity at balance load increased from about -1% with 32 MPa concrete to about -8% with 90 MPa concrete.

5. Conclusions

The stress-strain relationships published by Collins et al.(6) (Collins), and the curve specified in Eurocode 2 (4) for non-linear structural analysis (EC2-NL) are based on the behaviour of unconfined concrete and have steeply descending paths after the peak stress is reached. The Eurocode 2 parabolic-rectangle curve (EC2-P) has a constant stress from the peak stress up to the limiting strain, but the strain range used in the rectangular part of the curve is reduced for high strength grades. The Collins curve was found to be conservative compared with the EC2-P curve with 32 MPa concrete, but with relatively greater capacity for higher strength grades where the strain increment between the peak stress and the limiting strain is much less, with a reasonably close match between the two at 65 MPa, and a maximum moment capacity about 5% higher than the EC2-P curve at 90 MPa. The EC2-NL curve followed the same trend, but with moment capacities about 8% greater than the Collins Curve at 32 MPa, reducing to 2% at 90 MPa. The EC2-P curve is therefore relatively more conservative with higher strength grades, which is appropriate because of the reduced ductility of high strength concrete. The EC2-P curve has therefore been used as the benchmark for comparison with the rectangular stress blocks.

The rectangular stress block equivalent to the EC2-P curve (Equ-R) was found to give almost exactly the same results as the EC2-P curve with the rectangular cross section. This is to be expected because the Equ-R parameters were determined so that the average stress and the lever arm for the two distributions would be equal for a constant width compression zone. The minor differences were due to different stress corrections for the reinforcement in the compression zone. The Equ-R stress block was found to be up to 9% unconservative for the circular section, when compared with the EC2-P results, and up to 8% conservative for the I-section. The maximum difference was with Grade 90 concrete, the differences between the results being negligible at 32 MPa.

The Eurocode 2 rectangular stress block (EC2-R) surprisingly did not correlate well with the EC2-P curve. With a rectangular section the EC2-R stress block gave the least conservative results with 32 MPa concrete, and the most conservative with 90 MPa concrete.

The ACI 318 rectangular stress block gave almost identical results to AS 3600 with 32 MPa concrete, but became unconservative for concrete strengths of 65 MPa and greater. The 90 MPa results were significantly less conservative than any of the other curves, especially with the circular section.

With the rectangular cross section, the AS 3600 rectangular stress block was a little more conservative than the EC2-P curve at 32 MPa, and very close over the full range of axial loads at 65 MPa and 90 MPa. With the other sections it followed the same trend as the Equ-R stress block, being less conservative than the EC2-P curve with the circular section and higher strength grades, and more conservative with the I-section.

In summary, a rectangular stress block derived on a rational basis (the Equ-R stress block) gave almost identical results to the Eurocode 2 Parabolic-Rectangular curve for all concrete strengths when used with a rectangular section. The AS 3600 rectangular stress block gave almost identical results for the higher concrete strengths, and slightly more conservative results with 32 MPa concrete, There is therefore no great benefit in replacing the AS 3600 rectangular stress block with a more complex stress-strain curve for use with a rectangular concrete section.

When used with non-rectangular sections however, all the rectangular stress blocks had significant variations from the results of the EC2-P curve, being significantly less conservative with the circular section, and significantly more conservative with the I section. For this reason Eurocode 2 Cl 3.1.7(3) recommends that the concrete stress be reduced by 10% when the width of the compression zone decreases in the direction of the extreme compression fibre (4). Since there is little additional computational complexity in using a parabolic-linear stress block with non-rectangular sections it is recommended that the Eurocode 2 parabolic-linear curve be adopted for all non-rectangular sections. For compliance with AS 3600 Clause 3.1.4 the maximum stress must be taken as the characteristic cylinder strength factored by 0.9. If it is desired to use the same curve with rectangular sections the parabolic-linear curve may be conveniently converted to an equivalent rectangular stress block, which will generate almost identical results. This procedure gave more consistent results than adopting the Eurocode 2 rectangular stress block.

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